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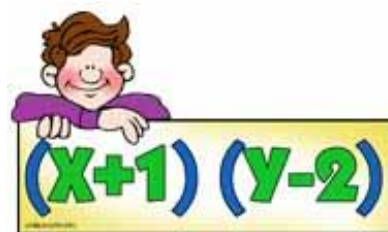
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# FACTORING QUADRATICS – THE REAL DEAL

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Hopefully you have grasped the concept of factoring (see Factoring – Introduction to Factoring Quadratics).



We now present a series of examples that try to turn the random method into something a little more systematic. But no matter what method is used, it's essentially a matter of *trial and error*. (Some teachers call it *guess and check*.)

## □ EXAMPLES

EXAMPLE 1:      **Factor:**  $6x^2 - 7x - 5$

Solution:    We first consider how  $6x^2$  can be broken up using multiplication. We see  $6x \cdot x$  and  $3x \cdot 2x$ .

Using  $6x$  and  $x$ , we start with

$$(6x \quad \quad)(x \quad \quad)$$

Now put numbers whose product is  $-5$  in the slots:

$$(6x + 5)(x - 1) = 6x^2 - 6x + 5x - 5 = 6x^2 - x - 5 \quad \text{☹}$$

$$(6x - 5)(x + 1) = 6x^2 + 6x - 5x - 5 = 6x^2 + x - 5 \quad \text{☹}$$

$$(6x + 1)(x - 5) = 6x^2 - 30x + x - 5 = 6x^2 - 29x - 5 \quad \text{☹}$$

$$(6x - 1)(x + 5) = 6x^2 + 30x - x - 5 = 6x^2 + 29x - 5 \quad \text{☹}$$

That's it for the  $-5$ ; we've tried everything.

Now we'll start again, using  $3x$  and  $2x$  in the front slots:

$$(3x \quad \quad)(2x \quad \quad)$$

Now try the same combinations as above for the  $-5$ :

$$(3x + 5)(2x - 1) = 6x^2 - 3x + 10x - 5 = 6x^2 + 7x - 5 \quad \text{☹}$$

$$(3x - 5)(2x + 1) = 6x^2 + 3x - 10x - 5 = \underline{6x^2 - 7x - 5} \quad \text{😊}$$

Although there are other ways to arrange the 5 and the 1, there's no need to go further; we've found what we were looking for.

Therefore, the final factorization of  $6x^2 - 7x - 5$  is

$(3x - 5)(2x + 1)$

**EXAMPLE 2:      Factor:  $n^2 - 5n + 6$**

**Solution:** As before, we focus on the first and last terms. There's only one way to break up the  $n^2$ , namely  $n \cdot n$ . So we start with

$$(n \quad \quad)(n \quad \quad)$$

But there are a few ways to break up the 6. Noting that 6 is positive — and therefore the signs of its factors must be the same — let's begin experimenting:

$$(n + 6)(n + 1) = n^2 + n + 6n + 6 = n^2 + 7n + 6 \quad \text{☹}$$

$$(n + 1)(n + 6) = n^2 + 6n + n + 6 = n^2 + 7n + 6 \quad \text{☹}$$

$$(n - 6)(n - 1) = n^2 - n - 6n + 6 = n^2 - 7n + 6 \quad \text{☹}$$

$$(n - 1)(n - 6) = n^2 - 6n - n + 6 = n^2 - 7n + 6 \quad \text{☹}$$

$$(n + 3)(n + 2) = n^2 + 2n + 3n + 6 = n^2 + 5n + 6 \quad \text{☹}$$

$$(n + 2)(n + 3) = n^2 + 3n + 2n + 6 = n^2 + 5n + 6 \quad \text{☹}$$

$$(n - 3)(n - 2) = n^2 - 2n - 3n + 6 = \underline{n^2 - 5n + 6} \quad \text{😊}$$

and thus our final factorization is

$$(n - 3)(n - 2)$$

Did we really need all those attempts to find the answer? NO.

Isn't it possible to see that the factors of 6 must be negative, considering the  $-5n$  term? OF COURSE.

You only need to use as many options as needed until you reach the required goal. Some students can just "see" what combination works, while others need a lot of experimenting. The more you practice, the quicker you'll find the right factors.

**EXAMPLE 3:**                      **Factor:**  $y^2 + 7y + 14$

**Solution:** There's really only one way to begin; we split the  $y^2$  term into  $y$  and  $y$ :

$$(y \quad \quad)(y \quad \quad)$$

Now let's work on the 14:

$$(y + 14)(y + 1) = y^2 + y + 14y + 14 = y^2 + 15y + 14 \quad \textcircled{\times}$$

$$(y + 1)(y + 14) = y^2 + 14y + y + 14 = y^2 + 15y + 14 \quad \textcircled{\times}$$

$$(y - 14)(y - 1) = y^2 - y - 14y + 14 = y^2 - 15y + 14 \quad \textcircled{\times}$$

$$(y - 1)(y - 14) = y^2 - 14y - y + 14 = y^2 - 15y + 14 \quad \textcircled{\times}$$

$$(y + 7)(y + 2) = y^2 + 2y + 7y + 14 = y^2 + 9y + 14 \quad \textcircled{\times}$$

$$(y + 2)(y + 7) = y^2 + 7y + 2y + 14 = y^2 + 9y + 14 \quad \textcircled{\times}$$

$$(y - 7)(y - 2) = y^2 - 2y - 7y + 14 = y^2 - 9y + 14 \quad \textcircled{\times}$$

$$(y - 2)(y - 7) = y^2 - 7y - 2y + 14 = y^2 - 9y + 14 \quad \textcircled{\times}$$

Nothing but sad faces, and we've tried every possible arrangement. This can mean only one thing: There's no way to factor the expression  $y^2 + 7y + 14$ . The expression is

$y^2 + 7y + 14$  is called **prime** because, like the prime number 13, it can't be factored.

Not factorable

**EXAMPLE 4:**      **Factor:**  $w^2 - 25$

**Solution:** This quadratic expression has only two terms, but which one is missing? When a quadratic starts with  $w^2$ , we expect the next term to contain a  $w$ . So it's the middle term that is missing. But the middle term is not the one we focus on anyway, so let's begin the usual process:

The binomial  $w^2 - 25$  is called a **difference of squares**, because it's  $w$ -squared minus 5-squared.

$$(w + 25)(w - 1) = w^2 - w + 25w - 25 = w^2 + 24w - 25$$

☹

$$(w - 25)(w + 1) = w^2 + w - 25w - 25 = w^2 - 24w - 25 \quad ☹$$

$$(w + 1)(w - 25) = w^2 - 25w + w - 25 = w^2 - 24w - 25 \quad ☹$$

$$(w - 1)(w + 25) = w^2 + 25w - w - 25 = w^2 + 24w - 25 \quad ☹$$

$$(w + 5)(w - 5) = w^2 - 5w + 5w - 25 = \underline{w^2 - 25} \quad 😊$$

Therefore, the expression  $w^2 - 25$  factors into

$$(w + 5)(w - 5)$$

**EXAMPLE 5:**      **Factor:  $9u^2 + 12u + 4$**

**Solution:**    The quadratic term,  $9u^2$ , can be broken down two ways,  $9u \times u$  and  $3u \times 3u$ .

Let's start the multiplications using  $9u$  and  $u$ :

$$(9u + 4)(u + 1) = 9u^2 + 9u + 4u + 4 = 9u^2 + 13u + 4 \quad \ominus$$

$$(9u + 1)(u + 4) = 9u^2 + 36u + u + 4 = 9u^2 + 37u + 4 \quad \ominus$$

$$(9u + 2)(u + 2) = 9u^2 + 18u + 2u + 4 = 9u^2 + 20u + 4 \quad \ominus$$

That's about it for the  $9u$  and  $u$  combination. Now for  $3u$  and  $3u$ :

$$(3u + 4)(3u + 1) = 9u^2 + 3u + 12u + 4 = 9u^2 + 15u + 4 \quad \ominus$$

$$(3u + 1)(3u + 4) = 9u^2 + 12u + 3u + 4 = 9u^2 + 15u + 4 \quad \ominus$$

$$(3u + 2)(3u + 2) = 9u^2 + 6u + 6u + 4 = \underline{9u^2 + 12u + 4} \quad \text{😊}$$

Eureka! The factorization of  $9u^2 + 12u + 4$  is  $(3u + 2)(3u + 2)$ , which we can write more succinctly as

$(3u + 2)^2$

**EXAMPLE 6:**      **Factor:  $a^2 + 49$**

**Solution:**    Let's start experimenting right away:

$$(a + 7)(a + 7) = a^2 + 7a + 7a + 49 = a^2 + 14a + 49 \quad \ominus$$

$$(a + 49)(a + 1) = a^2 + a + 49a + 49 = a^2 + 50a + 49 \quad \ominus$$

$$(a + 1)(a + 49) = a^2 + 49a + a + 49 = a^2 + 50a + 49 \quad \ominus$$

$$(a - 7)(a - 7) = a^2 - 7a - 7a + 49 = a^2 - 14a + 49 \quad \ominus$$

$$(a - 49)(a - 1) = a^2 - a - 49a + 49 = a^2 - 50a + 49 \quad \ominus$$

$$(a - 1)(a - 49) = a^2 - 49a - a + 49 = a^2 - 50a + 49 \quad \ominus$$

Have we tried everything? It appears we have, and no smiley face! (As we saw before, not every expression can be factored.) So we say that  $a^2 + 49$  is

Not factorable

**First Notice:** Even though  $a^2 + 49$  is not factorable,  $a^2 - 49$  is factorable, since  $a^2 - 49 = (a + 7)(a - 7)$ . The difference between a plus sign and a minus sign makes all the difference in the world — so be careful!

**Second Notice:** Some students jump to the conclusion that when two terms are connected by a plus sign, the expression is not factorable. But consider  $4x^2 + 16$ . It may not factor with two sets of parentheses like we're learning in this chapter, but it does have a greatest common factor (GCF) of 4, which can be factored out to produce  $4(x^2 + 4)$ . Thus,  $4x^2 + 16$  is factorable.

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## Homework

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1. Factor each expression:

- |                      |                    |                      |
|----------------------|--------------------|----------------------|
| a. $2x^2 + 3x + 1$   | b. $3n^2 - 7n + 2$ | c. $5a^2 + 3a - 2$   |
| d. $3m^2 - 11m - 20$ | e. $4x^2 - 3x - 1$ | f. $6u^2 + 7u - 10$  |
| g. $4z^2 - 4z - 3$   | h. $6y^2 - 5y - 6$ | i. $7n^2 - 45n + 18$ |
| j. $a^2 + 16a + 63$  | k. $2x^2 + x - 3$  | l. $7z^2 - 12z + 5$  |

2. Factor each expression:

- |                      |                     |                     |
|----------------------|---------------------|---------------------|
| a. $x^2 + 5x + 6$    | b. $x^2 - 5x + 6$   | c. $x^2 - 5x - 6$   |
| d. $x^2 + 5x - 6$    | e. $n^2 + 10n + 9$  | f. $z^2 - 4z - 5$   |
| g. $t^2 - 20t + 96$  | h. $u^2 - 6u - 16$  | i. $Q^2 + 34Q - 72$ |
| j. $x^2 + 25x + 156$ | k. $a^2 - 17a + 70$ | l. $t^2 + t - 110$  |

3. Factor each expression:

- |                      |                     |                       |
|----------------------|---------------------|-----------------------|
| a. $x^2 + 8x + 16$   | b. $y^2 - 10y + 25$ | c. $a^2 + 18a + 81$   |
| d. $b^2 - 20b + 100$ | e. $4z^2 + 4z + 1$  | f. $9n^2 - 24n + 16$  |
| g. $25x^2 - 30x + 9$ | h. $x^2 + 6x + 36$  | i. $2t^2 + 33t + 100$ |
| j. $16a^2 + 8a + 1$  | k. $9u^2 - 12u + 4$ | l. $w^2 + 50w + 625$  |

4. Factor each expression:

- |                |               |                |               |
|----------------|---------------|----------------|---------------|
| a. $p^2 - 1$   | b. $c^2 - 4$  | c. $R^2 - 16$  | d. $z^2 - 36$ |
| e. $x^2 - 25$  | f. $y^2 - 81$ | g. $n^2 - 10$  | h. $w^2 + 16$ |
| i. $a^2 - 144$ | j. $e^2 - 72$ | k. $m^2 + 100$ | l. $W^2 - 1$  |
| m. $x^2 - 9$   | n. $a^2 - 49$ | o. $g^2 - 64$  | p. $y^2 + 4$  |

5. Factor each expression:

- |                 |                   |                  |                  |
|-----------------|-------------------|------------------|------------------|
| a. $4x^2 - 9$   | b. $9y^2 - 49$    | c. $u^2 - 2$     | d. $v^2 + 1$     |
| e. $16z^2 - 49$ | f. $49w^2 - 16$   | g. $49a^2 - 144$ | h. $121b^2 - 64$ |
| i. $9x^2 + 2$   | j. $1 - x^2$      | k. $16 - n^2$    | l. $25 - 4g^2$   |
| m. $9 + t^2$    | n. $144N^2 - 169$ | o. $225a^2 - 1$  |                  |

6. Factor each expression:

- |                       |                       |                        |
|-----------------------|-----------------------|------------------------|
| a. $3x^2 + 10x - 8$   | b. $t^2 - 121$        | c. $y^2 + 10y + 25$    |
| d. $16a^2 - 121$      | e. $b^2 - 20$         | f. $n^2 + 121$         |
| g. $x^2 + 3x + 1$     | h. $12q^2 - 23q + 5$  | i. $6a^2 - 13a + 6$    |
| j. $x^2 + 14x + 13$   | k. $4y^2 - 49$        | l. $9Q^2 + 12Q + 4$    |
| m. $25z^2 - 10z + 1$  | n. $16x^2 + 34x - 15$ | o. $16x^2 + 118x - 15$ |
| p. $16x^2 - 77x - 15$ | q. $16x^2 - 72x + 45$ | r. $16a^2 - 8a + 1$    |
| s. $x^2 + 7x + 5$     | t. $8c^2 + 2c - 21$   | u. $8c^2 - 13c - 21$   |
| v. $3a^2 - 5a - 12$   | w. $6x^2 + 17x - 14$  | x. $2y^2 + 7y - 9$     |

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## Review Problems

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7. Factor each expression:

a.  $x^2 + 17x + 72$

b.  $y^2 - 9y + 8$

c.  $N^2 + 100$

d.  $N^2 - 100$

e.  $x^2 - 18x + 81$

f.  $a^2 + 10a + 25$

g.  $t^2 + 4t - 45$

h.  $a^2 - 21a - 22$

i.  $a^2 - 9a - 22$

j.  $2x^2 - 9x - 5$

k.  $6x^2 + x - 40$

l.  $6x^2 - 13x - 15$

m.  $6x^2 + 11x - 17$

n.  $R^2 - 144$

o.  $25n^2 - 30n + 9$

p.  $T^2 + 144$

q.  $49w^2 + 70w + 25$

r.  $9a^2 - 9a + 2$

s.  $9a^2 - 19a + 2$

t.  $x^2 + 16x - 36$

u.  $x^2 + 37x + 36$

v.  $30c^2 - 11c + 1$

w.  $16a^2 - 6a - 1$

x.  $36q^2 + 60q + 25$

y.  $x^2 + 8x + 18$

z.  $32n^2 - 66n + 27$

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## Solutions

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1. a.  $(2x + 1)(x + 1)$

b.  $(3n - 1)(n - 2)$

c.  $(5a - 2)(a + 1)$

d.  $(3m + 4)(m - 5)$

e.  $(4x + 1)(x - 1)$

f.  $(6u - 5)(u + 2)$

g.  $(2z + 1)(2z - 3)$

h.  $(2y - 3)(3y + 2)$

i.  $(7n - 3)(n - 6)$

j.  $(a + 9)(a + 7)$

k.  $(2x + 3)(x - 1)$

l.  $(7z - 5)(z - 1)$



2. a.  $(x + 3)(x + 2)$       b.  $(x - 2)(x - 3)$       c.  $(x - 6)(x + 1)$   
d.  $(x + 6)(x - 1)$       e.  $(n + 9)(n + 1)$       f.  $(z - 5)(z + 1)$   
g.  $(t - 12)(t - 8)$       h.  $(u - 8)(u + 2)$       i.  $(Q + 36)(Q - 2)$   
j.  $(x + 13)(x + 12)$       k.  $(a - 10)(a - 7)$       l.  $(t + 11)(t - 10)$
3. a.  $(x + 4)^2$       b.  $(y - 5)^2$       c.  $(a + 9)^2$   
d.  $(b - 10)^2$       e.  $(2z + 1)^2$       f.  $(3n - 4)^2$   
g.  $(5x - 3)^2$       h. Not factorable      i.  $(2t + 25)(t + 4)$   
j.  $(4a + 1)^2$       k.  $(3u - 2)^2$       l.  $(w + 25)^2$
4. a.  $(p + 1)(p - 1)$       b.  $(c + 2)(c - 2)$       c.  $(R + 4)(R - 4)$   
d.  $(z + 6)(z - 6)$       e.  $(x + 5)(x - 5)$       f.  $(y + 9)(y - 9)$   
g. Not factorable      h. Not factorable      i.  $(a + 12)(a - 12)$   
j. Not factorable      k. Not factorable      l.  $(W + 1)(W - 1)$   
m.  $(x + 3)(x - 3)$       n.  $(a + 7)(a - 7)$       o.  $(g + 8)(g - 8)$   
p. Not factorable
5. a.  $(2x + 3)(2x - 3)$       b.  $(3y + 7)(3y - 7)$       c. Not factorable  
d. Not factorable      e.  $(4z + 7)(4z - 7)$       f.  $(7w + 4)(7w - 4)$   
g.  $(7a + 12)(7a - 12)$       h.  $(11b + 8)(11b - 8)$       i. Not factorable  
j.  $(1 + x)(1 - x)$       k.  $(4 + n)(4 - n)$       l.  $(5 + 2g)(5 - 2g)$   
m. Not factorable      n.  $(12N + 13)(12N - 13)$       o.  $(15a + 1)(15a - 1)$
6. a.  $(3x - 2)(x + 4)$       b.  $(t + 11)(t - 11)$       c.  $(y + 5)^2$   
d.  $(4a + 11)(4a - 11)$       e. Not factorable      f. Not factorable  
g. Not factorable      h.  $(3q - 5)(4q - 1)$       i.  $(2a - 3)(3a - 2)$   
j.  $(x + 1)(x + 13)$       k.  $(2y + 7)(2y - 7)$       l.  $(3Q + 2)^2$   
m.  $(5z - 1)^2$       n.  $(8x - 3)(2x + 5)$       o.  $(8x - 1)(2x + 15)$   
p.  $(16x + 3)(x - 5)$       q.  $(4x - 3)(4x - 15)$       r.  $(4a - 1)^2$

- |                        |                        |                       |
|------------------------|------------------------|-----------------------|
| s. Not factorable      | t. $(4c + 7)(2c - 3)$  | u. $(8c - 21)(c + 1)$ |
| v. $(3a + 4)(a - 3)$   | w. $(2x + 7)(3x - 2)$  | x. $(2y + 9)(y - 1)$  |
| 7. a. $(x + 9)(x + 8)$ | b. $(y - 1)(y - 8)$    | c. Not factorable     |
| d. $(N + 10)(N - 10)$  | e. $(x - 9)^2$         | f. $(a + 5)^2$        |
| g. $(t + 9)(t - 5)$    | h. $(a - 22)(a + 1)$   | i. $(a - 11)(a + 2)$  |
| j. $(2x + 1)(x - 5)$   | k. $(3x + 8)(2x - 5)$  | l. $(6x + 5)(x - 3)$  |
| m. $(6x + 17)(x - 1)$  | n. $(R + 12)(R - 12)$  | o. $(5n - 3)^2$       |
| p. Not factorable      | q. $(7w + 5)^2$        | r. $(3a - 1)(3a - 2)$ |
| s. $(9a - 1)(a - 2)$   | t. $(x + 18)(x - 2)$   | u. $(x + 36)(x + 1)$  |
| v. $(5c - 1)(6c - 1)$  | w. $(8a + 1)(2a - 1)$  | x. $(6q + 5)^2$       |
| y. Not factorable      | z. $(16n - 9)(2n - 3)$ |                       |

“The school is the last expenditure on which America should be willing to economize.”

*Franklin D. Roosevelt*

